Statistics of thermal to shot noise crossover in chaotic cavities

Vladimir Al Osipov[1] and Eugene Kanzieper[2]

[1] Fachbereich Physik, Universität Duisburg-Essen, Duisburg D-47057, Germany
[2] Department of Applied Mathematics, H.I.T.—Holon Institute of Technology, Holon 58102, Israel

E-mail: vladimir.osipov@uni-due.de and eugene.kanzieper@hit.ac.il

Abstract

Recently formulated integrable theory of quantum transport (Osipov and Kanzieper, 2008 *Phys. Rev. Lett.* **101** 176804) is extended to describe sample-to-sample fluctuations of the noise power in chaotic cavities with broken time-reversal symmetry. Concentrating on the universal transport regime, we determine dependence of the noise power cumulants on the temperature, applied bias voltage and the number of propagating modes in the leads. Intrinsic connection between statistics of thermal to shot noise crossover and statistics of Landauer conductance is revealed and briefly discussed.

PACS numbers: 73.23.−b, 05.45.Mt, 02.30.Ik

1. Introduction: thermal versus shot noise

The charge transfer through a phase-coherent cavity exhibiting chaotic classical dynamics is a random process influenced by discreteness of the electron charge e and the quantum nature of electrons (Blanter and Büttiker 2000, Imry 2002, Martin 2005). Fluctuations of charge transmitted during a fixed time interval or, equivalently, fluctuations $\delta I(t)$ of current around its mean are quantified by the noise power

$$\mathcal{P} = 2 \int_{-\infty}^{+\infty} dt \, \langle \delta I(t + t_0) \delta I(t_0) \rangle_{t_0}, \tag{1.1}$$

where the brackets $\langle \cdots \rangle_{t_0}$ indicate the time averaging.

At temperatures $\theta = k_B T$ which are much larger than a bias voltage $\upsilon = eV$ applied to the cavity ($\theta \gg \upsilon$), the current fluctuations are dominated by the equilibrium *thermal noise*, also known as Johnson–Nyquist noise. Caused by fluctuating occupation numbers in a flow of carriers injected into cavity from electronic reservoirs, thermal noise extends over all frequencies up to the quantum limit θ/h. In the absence of electron–electron interactions,

its power at zero bias voltage ($\upsilon = 0$) is related to the scattering matrix $\mathcal{S}$ of the system composed of the cavity and the leads (Khlus 1987, Lesovik 1989, Büttiker 1990, 1992, Martin and Landauer 1992):

$$\mathcal{P}_{\text{th}}(\theta) = 4\theta G_0 \text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger). \tag{1.2}$$

Here, $G_0 = e^2/h$ is the conductance quantum. The projection matrices $\mathcal{C}_{1,2}$ encode the information about particular cavity-lead geometry and will be specified later on.

In the opposite limit of low temperatures ($\theta \ll \upsilon$), the current fluctuations are still significant even though the flow of incident electrons is essentially noiseless. In this temperature regime, nonequilibrium current fluctuations (known as a *shot noise*) exist because of (i) the granularity of the electron charge e and (ii) the stochastic nature of electron scattering inside the cavity which splits the electron wave into two or more partial waves leaving the cavity through different exits. It is this 'uncertainty of not knowing where the electron came from and where it will go to' (Oberholzer *et al* 2002) that makes the transmitted charge to fluctuate. At zero temperature, the scattering matrix approach brings the shot noise power in the form

$$\mathcal{P}_{\text{shot}}(\upsilon) = 2\upsilon G_0 [\text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger) - \text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger)^2]. \tag{1.3}$$

At finite temperatures, both sources of noise are operative, the total noise $\mathcal{P}(\theta, \upsilon)$ being a complicated function of temperature and bias voltage[3]:

$$\mathcal{P}(\theta, \upsilon) = 4\theta G_0 \left(\text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger)^2 + \frac{\upsilon}{2\theta} \coth\left(\frac{\upsilon}{2\theta}\right) [\text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger) - \text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger)^2] \right). \tag{1.4}$$

Equation (1.4) suggests that the crossover from thermal noise $\mathcal{P}_{\text{th}}(\theta) = \mathcal{P}(\theta, 0)$ to shot noise $\mathcal{P}_{\text{shot}}(\upsilon) = \mathcal{P}(0, \upsilon)$ depends in a sensitive way on the scattering properties of the cavity and the leads incorporated in the scattering matrix $\mathcal{S}$. Since chaotic scattering of electrons inside the cavity induces fluctuations of $\mathcal{S}$-matrix (Blümel and Smilansky 1990), the noise power $\mathcal{P}(\theta, \upsilon)$ fluctuates, too.

So far, the thermal to shot noise crossover has only been studied at the level of *average* noise power. For the two-terminal scattering geometry comprising the cavity attached to outside reservoirs (kept at temperature θ) via two leads supporting N_L and N_R propagating modes, respectively, the average noise power equals (Blanter and Sukhorukov 2000, Oberholzer *et al* 2001, Savin and Sommers 2006)

$$\langle \mathcal{P}(\theta, \upsilon) \rangle_{\mathcal{S}} = \langle \mathcal{P}_{\text{th}} \rangle_{\mathcal{S}} \left[1 + \frac{N_\text{L} N_\text{R}}{(N_\text{L} + N_\text{R})^2 - 1} f_\beta \right], \tag{1.5}$$

where

$$\langle \mathcal{P}_{\text{th}} \rangle_{\mathcal{S}} = 4\theta G_0 \frac{N_\text{L} N_\text{R}}{N_\text{L} + N_\text{R}} \tag{1.6}$$

is the average equilibrium thermal noise power, and the thermodynamic function

$$f_\beta = \beta \coth \beta - 1 \tag{1.7}$$

depends on the ratio $\beta = \upsilon/2\theta$ between the bias voltage υ and the temperature θ. Equations (1.5) and (1.6) hold for cavities with broken time reversal symmetry.[4] Derived for the universal transport regime (Beenakker 1997, Richter and Sieber 2002, Müller *et al* 2007) emerging in the limit $\tau_\text{D} \gg \tau_\text{E}$ (Agam *et al* 2000), where τ_D is the average electron dwell time and τ_E is the Ehrenfest time (the time scale where quantum effects set in), the above prediction

---

[3] Equation (1.4) disregards the low-frequency $1/f$ noise that can efficiently be filtered out in experiments.

[4] The two can readily be extended to other symmetry classes, see Savin and Sommers (2006).

has been confirmed in a remarkable series of experiments (Oberholzer *et al* 2001, 2002, Cron *et al* 2001).

In this paper, we examine *statistics* of the thermal to shot noise crossover. The latter, contained in the distribution function of the noise power $\mathcal{P}(\theta, \upsilon)$ or, equivalently, in its *cumulants* $\langle\langle \mathcal{P}^\ell \rangle\rangle$, can effectively be described within the framework of integrable theory of quantum transport formulated by Osipov and Kanzieper (2008). Let us stress that recent experimental studies (Flindt *et al* 2009) of quantum noise fluctuations in nanoscale conductors (which concentrated on detection of higher cumulants of noise) suggest that testing our predictions may be feasible within the current limits of nanotechnology.

2. Integrable theory of noise power fluctuations

In what follows, we consider chaotic cavities with broken time-reversal symmetry which are probed, via ballistic point contacts, by two (left and right) leads; the leads supporting N_L and N_R propagating modes, respectively, are further coupled to external reservoirs kept at the temperature θ. This scattering geometry corresponds to the projection matrices $\mathcal{C}_{1,2}$ of the form

$$\mathcal{C}_1 = \begin{pmatrix} 1_{N_L} & 0 \\ 0 & 0_{N_R} \end{pmatrix}, \qquad \mathcal{C}_2 = \begin{pmatrix} 0_{N_L} & 0 \\ 0 & 1_{N_R} \end{pmatrix}, \tag{2.1}$$

see equations (1.2)–(1.4).

2.1. Joint cumulants of Landauer conductance and noise power

The starting point of our analysis is the *joint* cumulant generating function (JCGF)

$$\mathcal{F}_n(z, w) = \langle \exp(-zG/G_0) \exp(-w\mathcal{P}/\mathcal{P}_0) \rangle_{\mathcal{S}\in\text{CUE}(N)} \tag{2.2}$$

of the Landauer conductance $G = G_0 \text{tr}(\mathcal{C}_1 \mathcal{S} \mathcal{C}_2 \mathcal{S}^\dagger)$ and the noise power $\mathcal{P}(\theta, \upsilon)$ measured in the units of $G_0 = e^2/h$ and $\mathcal{P}_0 = 4\theta G_0$, respectively. The joint dimensionless cumulants

$$\kappa_{\ell,m} = \langle\langle (G/G_0)^\ell (\mathcal{P}/\mathcal{P}_0)^m \rangle\rangle \tag{2.3}$$

can be extracted from the expansion

$$\log \mathcal{F}_n(z, w) = \sum_{\ell,m=0}^{\infty} (-1)^{\ell+m} \frac{z^\ell w^m}{\ell! m!} \kappa_{\ell,m}, \tag{2.4}$$

where $\kappa_{0,0} \equiv 0$. In both equations (2.2) and (2.4), the subscript n stands for $n = \min(N_L, N_R)$, and $N = N_L + N_R$ is the total number of propagating modes (channels) in the leads. The notation $\mathcal{S} \in \text{CUE}(N)$ indicates that averaging runs over scattering matrices $\mathcal{S}$ drawn from the Dyson circular unitary ensemble (Blümel and Smilansky 1990, Mello and Baranger 1999, Mehta 2004). The latter is microscopically justified (Lewenkopf and Weidenmüller 1991, Brouwer 1995) in the universal transport regime we are confined to.

To perform the averaging in equation (2.2) in a most economic way, we employ a polar decomposition (Hua 1963, Baranger and Mello 1994, Forrester 2006) of $\mathcal{S}$-matrix. Bringing into play a set of n transmission eigenvalues $\boldsymbol{T} = (T_1, \ldots, T_n) \in (0, 1)^n$ distributed in accordance with the joint probability density function

$$P_n(\boldsymbol{T}) = c_n^{-1} \Delta_n^2(\boldsymbol{T}) \prod_{j=1}^{n} T_j^\nu, \tag{2.5}$$

this decomposition highlights Landauer's idea of viewing conductance as transmission,

$$G(\boldsymbol{T}) = G_0 \sum_{j=1}^{n} T_j. \tag{2.6}$$

Simultaneously, it reduces the expression for noise power (equation (1.4)) down to

$$\mathcal{P}(\boldsymbol{T}) = \mathcal{P}_0 \left(\sum_{j=1}^{n} T_j + f_\beta \sum_{j=1}^{n} T_j(1 - T_j) \right). \tag{2.7}$$

The parameter ν in equation (2.5) is a measure of asymmetry between the leads, $\nu = |N_{\mathrm{L}} - N_{\mathrm{R}}|$, the notation $\Delta_n(\boldsymbol{T})$ stands for the Vandermonde determinant $\Delta_n(\boldsymbol{T}) = \prod_{j<k}(T_k - T_j)$, whilst c_n is a normalization constant. As the result, we are left with the JCGF in the form

$$\mathcal{F}_n(z, w) = c_n^{-1} \int_{(0,1)^n} \prod_{j=1}^{n} \mathrm{d}T_j T_j^\nu \Gamma_{z,w}(T_j) \Delta_n^2(\boldsymbol{T}), \tag{2.8}$$

where

$$\Gamma_{z,w}(T) = \exp[-(z + w)T - wf_\beta T(1 - T)]. \tag{2.9}$$

Although the above matrix integral representation of the JCGF $\mathcal{F}_n(z, w)$ is by far more complicated than the one appearing in the integrable theory of conductance fluctuations (Osipov and Kanzieper 2008),

$$\mathcal{F}_n(z, 0) = \langle \exp(-zG/G_0) \rangle_{\mathcal{S} \in \mathrm{CUE}(N)} = c_n^{-1} \int_{(0,1)^n} \prod_{j=1}^{n} \mathrm{d}T_j T_j^\nu \mathrm{e}^{-zT_j} \Delta_n^2(\boldsymbol{T}), \tag{2.10}$$

it can still be treated nonperturbatively, much in line with the formalism used in the exact approach to zero-dimensional replica sigma models (Kanzieper 2002, 2005, 2009, Osipov and Kanzieper 2007).

2.2. The τ function theory of the joint cumulant generating function

The 'deform-and-study' approach (Morozov 1994, Adler *et al* 1995, Adler and van Moerbeke 2001) borrowed from the theory of integrable systems is central to the nonperturbative calculation of $\mathcal{F}_n(z, w)$. In the present context, the main idea of the method consists of 'embedding' $\mathcal{F}_n(z, w)$ into a more general theory of the τ function

$$\tau_n(\boldsymbol{t}; z, w) = \frac{1}{n!} \int_{(0,1)^n} \prod_{j=1}^{n} \mathrm{d}T_j T_j^\nu \Gamma_{z,w}(T_j) \mathrm{e}^{V(\boldsymbol{t}; T_j)} \Delta_n^2(\boldsymbol{T}) \tag{2.11}$$

which possesses the infinite-dimensional parameter space $\boldsymbol{t} = (t_1, t_2, \ldots)$ arising as the result of the $\boldsymbol{t}$ deformation

$$V(\boldsymbol{t}; T) = \sum_{k=1}^{\infty} t_k T^k. \tag{2.12}$$

Studying an evolution of the τ function in the extended $(n, \boldsymbol{t}, z, w)$ space allows us to identify various nonlinear differential hierarchical relations. A projection of these relations onto the hyperplane $\boldsymbol{t} = \boldsymbol{0}$,

$$\mathcal{F}_n(z, w) = \frac{n!}{c_n} \tau_n(\boldsymbol{t}; z, w) \Big|_{\boldsymbol{t}=\boldsymbol{0}}, \tag{2.13}$$

generates, among others, a closed nonlinear differential equation for the JCGF $\mathcal{F}_n(z, w)$. It is this equation (2.24) that will further supply the cumulants of noise power.

The two key ingredients of the exact theory of τ functions are (i) the bilinear identity (Date *et al* 1983) and (ii) the (linear) Virasoro constraints (Mironov and Morozov 1990).

2.2.1. Bilinear identity and Kadomtsev–Petviashvili equation. The bilinear identity encodes an infinite set of hierarchically structured nonlinear differential equations in the variables $\boldsymbol{t} = (t_1, t_2, \ldots)$. For the model introduced in equation (2.11), the bilinear identity reads (Adler *et al* 1995, Tu *et al* 1996):

$$\oint_{\mathcal{C}_\infty} \mathrm{d}z \, \mathrm{e}^{av(t-t';z)} \tau_n(\boldsymbol{t} - [\boldsymbol{z}^{-1}]) \frac{\tau_{m+1}(\boldsymbol{t}' + [\boldsymbol{z}^{-1}])}{z^{m+1-n}}$$
$$= \oint_{\mathcal{C}_\infty} \mathrm{d}z \, \mathrm{e}^{(a-1)v(t-t';z)} \tau_m(\boldsymbol{t}' - [\boldsymbol{z}^{-1}]) \frac{\tau_{n+1}(\boldsymbol{t} + [\boldsymbol{z}^{-1}])}{z^{n+1-m}}. \tag{2.14}$$

Here, $a \in \mathbb{R}$ is a free parameter; the integration contour $\mathcal{C}_\infty$ encompasses the point $z = \infty$; the notation $\boldsymbol{t} \pm [\boldsymbol{z}^{-1}]$ stands for the infinite set of parameters $\{t_j \pm z^{-j}/j\}$; for brevity, both z and w were dropped from the arguments of τ functions.

Being expanded in terms of $\boldsymbol{t}' - \boldsymbol{t}$ and a, equation (2.14) generates a variety of integrable hierarchies (Osipov and Kanzieper 2009). One of them is the Kadomtsev–Petviashvili (KP) hierarchy. Its first nontrivial member

$$\left(\frac{\partial^4}{\partial t_1^4} + 3\frac{\partial^2}{\partial t_2^2} - 4\frac{\partial^2}{\partial t_1 \partial t_3} \right) \log \tau_n(\boldsymbol{t}; z, w) + 6\left(\frac{\partial^2}{\partial t_1^2} \log \tau_n(\boldsymbol{t}; z, w) \right)^2 = 0 \tag{2.15}$$

is of primary importance since its projection onto $\boldsymbol{t} = \boldsymbol{0}$ (equation (2.13)) gives rise to a nonlinear differential equation for the JCGF $\mathcal{F}_n(z, w)$. The resulting equation will further be used to determine the noise power cumulants we are aimed at.

2.2.2. Virasoro constraints. Since we are interested in deriving a differential equation for $\mathcal{F}_n(z, w)$ in terms of the derivatives over variables z and w, we have to seek an additional block of the theory that would make a link between t_j-derivatives in equation (2.15) taken at $\boldsymbol{t} = \boldsymbol{0}$ and the derivatives over w and z. This missing block is the *Virasoro constraints* which reflect the invariance of the τ function (equation (2.11)) under a change of the integration variables.

In the present context, it is useful to demand the invariance under the set of transformations

$$T_j \to \tilde{T}_j + \epsilon \tilde{T}_j^{q+1}(\tilde{T}_j - 1), q \geqslant 0. \tag{2.16}$$

Employing by now a standard procedure (Mironov and Morozov 1990, Adler and van Moerbeke 1995), one readily checks that transformation (2.16) induces Virasoro constraints in the form

$$\left[\hat{L}_{q+1}(\boldsymbol{t}) - \hat{L}_q(\boldsymbol{t}) \right] \tau_n(\boldsymbol{t}; z, w) = 0, \qquad q \geqslant 0, \tag{2.17}$$

where a set of differential operators

$$\hat{L}_q(\boldsymbol{t}) = \hat{\mathcal{L}}_q(\boldsymbol{t}) + 2f_\beta w \frac{\partial}{\partial t_{q+2}} - [z + (1 + f_\beta)w] \frac{\partial}{\partial t_{q+1}} + \nu \frac{\partial}{\partial t_q} \tag{2.18}$$

involves the Virasoro operators

$$\hat{\mathcal{L}}_q(\boldsymbol{t}) = \sum_{j=1}^{\infty} j t_j \frac{\partial}{\partial t_{q+j}} + \sum_{j=0}^{q} \frac{\partial^2}{\partial t_j \partial t_{q-j}}, \tag{2.19}$$

satisfying the Virasoro algebra

$$[\hat{\mathcal{L}}_p, \hat{\mathcal{L}}_q] = (p - q)\hat{\mathcal{L}}_{p+q}, \qquad p, q \geqslant -1. \tag{2.20}$$

In equations (2.18) and (2.19), the convention $\partial/\partial t_0 \equiv n$ is assumed.

2.2.3. Nonlinear differential equation for $\mathcal{F}_n(z, w)$. To derive a differential equation for the JCGF $\mathcal{F}_n(z, w)$, one has to project the first KP equation (2.15) onto the hyperplane $\boldsymbol{t} = \boldsymbol{0}$. Spotting the identities ($f_\beta > 0$)

$$\frac{\partial}{\partial t_1} \tau_n(\boldsymbol{t}; z, w) = -\frac{\partial}{\partial z} \tau_n(\boldsymbol{t}; z, w), \tag{2.21}$$

$$f_\beta \frac{\partial}{\partial t_2} \tau_n(\boldsymbol{t}; z, w) = \frac{\partial}{\partial w} \tau_n(\boldsymbol{t}; z, w) - (1 + f_\beta) \frac{\partial}{\partial z} \tau_n(\boldsymbol{t}; z, w), \tag{2.22}$$

we combine equation (2.15) with the Virasoro constraints equation (2.17) taken at $q = 0$,

$$\left[\sum_{j=1}^{\infty} j t_j \left(\frac{\partial}{\partial t_{j+1}} - \frac{\partial}{\partial t_j} \right) + 2 f_\beta w \left(\frac{\partial}{\partial t_3} - \frac{\partial}{\partial t_2} \right) - [z + (1 + f_\beta) w] \left(\frac{\partial}{\partial t_2} - \frac{\partial}{\partial t_1} \right) \right.$$
$$\left. + (N_{\mathrm{L}} + N_{\mathrm{R}}) \frac{\partial}{\partial t_1} - N_{\mathrm{L}} N_{\mathrm{R}} \right] \tau_n(\boldsymbol{t}; z, w) = 0, \tag{2.23}$$

to derive

$$\left[w f_\beta^2 \frac{\partial^4}{\partial z^4} + \left[2(N_{\mathrm{L}} + N_{\mathrm{R}}) f_\beta - 2z + w (1 - f_\beta^2) \right] \frac{\partial^2}{\partial z^2} + 2(z - 2w) \frac{\partial^2}{\partial z \partial w} + 3w \frac{\partial^2}{\partial w^2} \right.$$
$$\left. + 2 \left(\frac{\partial}{\partial w} - \frac{\partial}{\partial z} \right) \right] \log \mathcal{F}_n(z, w) + 6 w f_\beta^2 \left(\frac{\partial^2}{\partial z^2} \log \mathcal{F}_n(z, w) \right)^2 = 0. \tag{2.24}$$

Owing to equation (2.4), this nonlinear equation considered *together* with the equation for $\mathcal{F}_n(z, 0)$ contains all the information about joint cumulants of the Landauer conductance and the noise power. The latter equation, written in terms of

$$\sigma_n(z) = N_{\mathrm{L}} N_{\mathrm{R}} + z \frac{\partial}{\partial z} \log \mathcal{F}_n(z, 0), \tag{2.25}$$

reads (Osipov and Kanzieper 2008)

$$z^2 \frac{\partial^3}{\partial z^3} \sigma_n(z) + z \frac{\partial^2}{\partial z^2} \sigma_n(z) + 6z \left(\frac{\partial}{\partial z} \sigma_n(z) \right)^2 - 4 \sigma_n \frac{\partial}{\partial z} \sigma_n(z)$$
$$- \left[(z - (N_{\mathrm{L}} + N_{\mathrm{R}}))^2 - 4 N_{\mathrm{L}} N_{\mathrm{R}} \right] \frac{\partial}{\partial z} \sigma_n(z) - (N_{\mathrm{L}} + N_{\mathrm{R}} - z) \sigma_n(z) = 0. \tag{2.26}$$

This can be recognized as the Chazy form (Chazy 1911, Cosgrove and Scoufis 1993) of the fifth Painlevé transcendent

$$\left(z \frac{\partial^2}{\partial z^2} \sigma_n(z) \right)^2 - \left[\sigma_n(z) + 2 \left(\frac{\partial}{\partial z} \sigma_n(z) \right)^2 + (N_{\mathrm{L}} + N_{\mathrm{R}} - z) \frac{\partial}{\partial z} \sigma_n(z) \right]^2$$
$$+ 4 \left(\frac{\partial}{\partial z} \sigma_n(z) \right)^2 \left(N_{\mathrm{L}} + \frac{\partial}{\partial z} \sigma_n(z) \right) \left(N_{\mathrm{R}} + \frac{\partial}{\partial z} \sigma_n(z) \right) = 0 \tag{2.27}$$

written in the Jimbo–Miwa–Okamoto form (Jimbo *et al* 1980, Okamoto 1987). For completeness, we have included a detailed derivation of equation (2.26) in appendix A.

2.3. Recurrence solution for joint cumulants

Combined with the cumulant expansion equation (2.4), the differential equation (2.24) furnishes the nonlinear recurrence for the joint dimensionless cumulants $\kappa_{\ell,m}$ of conductance and noise power ($\ell, m \geqslant 0$):

$$
\begin{aligned}
m\big[f_\beta^2 \kappa_{\ell+4,m-1} &+ (1-f_\beta^2)\kappa_{\ell+2,m-1}\big] - 2(N_L+N_R)f_\beta\kappa_{\ell+2,m} \\
&- 2(\ell+2m+1)\,\kappa_{\ell+1,m} + (2\ell+3m+2)\kappa_{\ell,m+1} \\
&+ 6mf_\beta^2 \sum_{i=0}^{m-1}\binom{m-1}{i}\sum_{j=0}^{\ell}\binom{\ell}{j}\kappa_{j+2,i}\kappa_{\ell-j+2,m-i-1} = 0.
\end{aligned}
\tag{2.28}
$$

To resolve it, one must know the boundary conditions whose rôle is played by cumulants $\kappa_{\ell,0} = \langle\!\langle (G/G_0)^\ell \rangle\!\rangle$ of the dimensionless Landauer conductance. These have been nonperturbatively calculated in our previous publication (Osipov and Kanzieper 2008). Indeed, given the mean conductance

$$
\kappa_{1,0} = \frac{N_L N_R}{N_L + N_R}
\tag{2.29}
$$

and its variance

$$
\kappa_{2,0} = \frac{\kappa_{1,0}^2}{(N_L+N_R)^2 - 1},
\tag{2.30}
$$

the higher order cumulants $\kappa_{\ell,0}$'s are determined by the one-dimensional recurrence[5]

$$
\begin{aligned}
[(N_L+N_R)^2 - \ell^2](\ell+1)\kappa_{\ell+1,0} &+ (N_L+N_R)(2\ell-1)\ell\kappa_{\ell,0} + \ell(\ell-1)(\ell-2)\kappa_{\ell-1,0} \\
&- 2\sum_{j=0}^{\ell-1}(3j+1)(j-\ell)^2\binom{\ell}{j}\kappa_{j+1,0}\kappa_{\ell-j,0} = 0.
\end{aligned}
\tag{2.31}
$$

Equations (2.28) and (2.31) represent the main result of our study[6]. They provide a nonperturbative description of the noise power fluctuations in the crossover region between the thermal and the shot noise (Savin *et al* 2008) regimes (as discussed in the Introduction) *by relating the temperature (θ) and bias–voltage (υ) dependent cumulants of the noise power to those of the Landauer conductance.*

---

[5] Equations (2.29), (2.30) and (2.31) follow from the cumulant expansion

$$
\log \mathcal{F}_n(z,0) = \sum_{\ell=1}^{\infty}(-1)^\ell \frac{z^\ell}{\ell!}\kappa_{\ell,0},
$$

see equations (2.4) and (2.10), substituted into equation (2.27). Successive iterations of equation (2.31) yield the cumulants $\kappa_{\ell,0}$ of Landauer conductance in the form

$$
\kappa_{\ell,0} = \frac{(\ell-1)!}{\prod_{j=1}^{\ell-1}(N^2 - j^2)} p_\ell(\kappa_{1,0}),
$$

where the first few polynomials $p_\ell(\kappa)$ are

$$
\begin{aligned}
p_1(\kappa) &= \kappa, \\
p_2(\kappa) &= \kappa^2, \\
p_3(\kappa) &= 4\kappa^3 - N\kappa^2, \\
p_4(\kappa) &= 12\left(2 - \frac{1}{N^2-1}\right)\kappa^4 - 10N\kappa^3 + (N^2+1)\kappa^2.
\end{aligned}
$$

Here, $N = N_L + N_R$.

[6] Note that equation (15) in the paper by Osipov and Kanzieper (2008) contains typos. The correct formula is given by equation (2.31).

Undoubtedly, the very existence of the above nontrivial relation (which emphasizes a fundamental rôle played by Landauer conductance in transport problems) must be well rooted in the mathematical formalism and also have a good physics reason. As far as the former point is concerned, we wish to stress that a naïve attempt to build a theory for the generating function $\mathcal{F}_n(0, w)$ of solely noise power cumulants faces an unsurmountable obstacle: the KP equation (equation (2.15)) and appropriate Virasoro constraints (equation (2.17) at $z = 0$) cannot be resolved jointly in the hyperplane $t = 0$. This justifies the starting point (equation (2.2)) of our analysis. The physics arguments behind the peculiar structure of our solution are yet to be found.

2.4. Noise power cumulants in the crossover regime

Some computational effort is needed to read off explicit formulae for the noise power cumulants from equations (2.28) and (2.31). Below we provide expressions for two families of joint cumulants expressed in terms of dimensionless cumulants $\kappa_{\ell,0} = \langle\langle (G/G_0)^\ell \rangle\rangle$ of the Landauer conductance.

- Mean noise power

$$\langle\langle \mathcal{P} \rangle\rangle = 4\theta G_0 [N f_\beta \kappa_{2,0} + \kappa_{1,0}] \tag{2.32}$$

  is generated by the lowest order member $(\ell, m) = (0, 0)$ of the recurrence equation (2.28). Being in concert with the known expression (equations (1.5) and (1.6)), this result is a particular case of a more general formula

$$\kappa_{\ell,1} = \kappa_{\ell+1,0} + N \frac{f_\beta}{\ell + 1} \kappa_{\ell+2,0}. \tag{2.33}$$

- Noise power variance,

$$\langle\langle \mathcal{P}^2 \rangle\rangle = (4\theta G_0)^2 \left[\left(\frac{2}{3} N^2 - 1 \right) \frac{f_\beta^2}{5} \kappa_{4,0} + N f_\beta \kappa_{3,0} + \left(1 + \frac{f_\beta^2}{5} \right) \kappa_{2,0} - \frac{6}{5} f_\beta^2 \kappa_{2,0}^2 \right], \tag{2.34}$$

  is supplied by the $(\ell, m) = (0, 1)$ member of the recurrence. Its generalization reads

$$\kappa_{\ell,2} = \left(\frac{2N^2}{\ell + 3} - 1 \right) \frac{f_\beta^2}{2\ell + 5} \kappa_{\ell+4,0} + 2N \frac{f_\beta}{\ell + 2} \kappa_{\ell+3,0} + \left(1 + \frac{f_\beta^2}{2\ell + 5} \right) \kappa_{\ell+2,0}$$

$$- 6 \frac{f_\beta^2}{2\ell + 5} \sum_{j=0}^{\ell} \binom{\ell}{j} \kappa_{j+2,0} \kappa_{\ell+2-j,0}. \tag{2.35}$$

  Here and above, $N = N_L + N_R$.

The noise power cumulants $\langle\langle \mathcal{P}^\ell \rangle\rangle$ of higher order ($\ell \geqslant 3$) can be calculated in the same manner albeit explicit expressions become increasingly cumbersome. Varying therein the parameters (θ, υ) from $(\theta, 0)$ to $(0, \upsilon)$, one observes a smooth crossover between the thermal and the shot noise regime.

2.5. Large-n analysis of joint cumulants: symmetric leads

The nonpeturbative solution (equations (2.28) and (2.31)) has a drawback: it does not provide much desired *explicit* dependence of conductance and/or noise power cumulants $\kappa_{\ell,m}$ on parameters of the scattering system. To probe such a dependence, we turn to the large-n limit

of the recurrence equation (2.28). In what follows, the asymmetry parameter ν will be set to zero.

Under the latter assumption ($\nu = 0$), the joint cumulants $\kappa_{\ell,m}$ are solutions to the recurrence equation ($\ell, m \geqslant 0$)

$$
m\big[f_\beta^2 \kappa_{\ell+4,m-1} + (1 - f_\beta^2)\kappa_{\ell+2,m-1}\big] - 4nf_\beta \kappa_{\ell+2,m}
$$
$$
- 2(\ell + 2m + 1)\kappa_{\ell+1,m} + (2\ell + 3m + 2)\kappa_{\ell,m+1}
$$
$$
+ 6mf_\beta^2 \sum_{i=0}^{m-1}\binom{m-1}{i}\sum_{j=0}^{\ell}\binom{\ell}{j}\kappa_{j+2,i}\kappa_{\ell-j+2,m-i-1} = 0 \tag{2.36}
$$

which must be supplemented by yet another recurrence ($\ell \geqslant 2$)

$$
(4n^2 - \ell^2)(\ell + 1)\kappa_{\ell+1,0} + 2n(2\ell - 1)\ell\kappa_{\ell,0} + \ell(\ell - 1)(\ell - 2)\kappa_{\ell-1,0}
$$
$$
- 2\sum_{j=0}^{\ell-1}(3j + 1)(j - \ell)^2\binom{\ell}{j}\kappa_{j+1,0}\kappa_{\ell-j,0} = 0 \tag{2.37}
$$

that brings, in turn, a set of initial conditions $\kappa_{\ell,0}$ to equation (2.36).

2.5.1. Cumulants of Landauer conductance. It is instructive to start with the asymptotic analysis of equation (2.37). In the case of symmetric leads, the conductance cumulants of odd order vanish[7], $\kappa_{2\ell+1,0} \equiv 0$ for all $\ell \geqslant 1$ albeit $\kappa_{1,0} = n/2$. As the result, one is left with the recurrence equation for the cumulants $\kappa_{2\ell,0}$ of even order ($\ell \geqslant 1$)

$$
[4n^2 - (2\ell + 1)^2](\ell + 1)\kappa_{2\ell+2,0} + \ell(4\ell^2 - 1)\kappa_{2\ell,0}
$$
$$
- 8\sum_{j=0}^{\ell-1}(3j + 2)(j - \ell)^2\binom{2\ell + 1}{2j + 1}\kappa_{2j+2,0}\kappa_{2\ell-2j,0} = 0 \tag{2.38}
$$

subject to the initial condition (equation (2.30))

$$
\kappa_{2,0} = \frac{n^2}{4(4n^2 - 1)} = \frac{1}{16}\sum_{\sigma=0}^{\infty}\frac{1}{(4n^2)^\sigma}. \tag{2.39}
$$

Since, in the limit of a large number of propagating modes ($n \gg 1$), the conductance distribution is expected to roughly follow the Gaussian law (Politzer 1989) with the mean $\kappa_{1,0}^{(G)} = n/2$ and the variance $\kappa_{2,0}^{(G)} = 1/16$ (see equations (2.29) and (2.30)), it is natural to seek a large-n solution to equation (2.38) in the form ($j \geqslant 1$)

$$
\kappa_{2\ell,0} = \frac{1}{16}\delta_{\ell,1} + \delta\kappa_{2\ell,0}, \tag{2.40}
$$

where $\delta\kappa_{2\ell,0}$ (with $\ell \geqslant 2$) account for deviations from the Gaussian distribution. Putting forward the large-n ansatz

$$
\delta\kappa_{2\ell,0} = \frac{1}{n^{2\ell}}\sum_{\sigma=0}^{\infty}\frac{a_{2\ell}(2\sigma)}{n^{2\sigma}}, \ell \geqslant 1, \tag{2.41}
$$

where (see equation (2.39))

$$
a_2(2\sigma) = \frac{1}{2^{2\sigma+4}}, \tag{2.42}
$$

---

[7] At the formal level, this is direct consequence of the identity $\mathcal{F}_n(z, 0) = e^{-nz}\mathcal{F}_n(-z, 0)$ holding as soon as $\nu = 0$, see equation (2.10).

we further substitute it into equation (2.38) to derive

$$\frac{a_{2\ell+2}(0)}{a_{2\ell}(0)} = \frac{1}{2^4} \frac{(2\ell+1)!}{(2\ell-1)!}, \qquad \ell \geqslant 2, \tag{2.43}$$

and

$$\frac{a_4(0)}{a_2(2)} = \frac{3!}{2^4}. \tag{2.44}$$

Hence,

$$a_{2\ell}(0) = \frac{1}{4} \frac{(2\ell-1)!}{4^{2\ell}}. \tag{2.45}$$

Taken together with equation (2.42), this yields the leading term in the $1/n$ expansion (equation (2.41)) for conductance cumulants:

$$\delta\kappa_{2\ell} \simeq \frac{1}{4} \frac{(2\ell-1)!}{(4n)^{2\ell}}. \tag{2.46}$$

The higher order corrections to equation (2.46) can be obtained in a regular way.

2.5.2. Dependence of the noise power cumulants on temperature and bias voltage.
Similarly to the previous subsection, we start an asymptotic analysis of the recurrence equation (2.36) with singling out the large-n Gaussian part:

$$\kappa_{\ell,m} = \frac{n}{2} \left[\delta_{\ell,1}\delta_{m,0} + \left(1 + \frac{f_\beta}{4}\right) \delta_{\ell,0}\delta_{m,1} \right]$$
$$+ \frac{1}{16} \left[\delta_{\ell,1}\delta_{m,1} + \delta_{\ell,2}\delta_{m,0} + \left(1 + \frac{f_\beta^2}{8}\right) \delta_{\ell,0}\delta_{m,2} \right] + \delta\kappa_{\ell,m}. \tag{2.47}$$

The Gaussian part was read off from equations (2.33) and (2.35); the term $\delta\kappa_{\ell,m}$ accommodates non-Gaussian corrections to the joint cumulants of Landauer conductance and the noise power. Their large-n behaviour can be studied within the $1/n$ ansatz

$$\delta\kappa_{\ell,m} = \frac{1}{n^{\ell+m}} \sum_{\sigma=0}^{\infty} \frac{a_{\ell,m}(\sigma)}{n^\sigma}, \tag{2.48}$$

where $a_{1,0}(\sigma) = 0$. Substitution of equations (2.47) and (2.48) into the two-dimensional recurrence equation (2.36) brings the recurrence equation ($\ell + m > 0$)

$$m\left(1 - \frac{f_\beta^2}{4}\right) a_{\ell+2,m-1}(0) - 4f_\beta a_{\ell+2,m}(0)$$
$$- 2(\ell+1+2m)a_{\ell+1,m}(0) + (2\ell+2+3m)a_{\ell,m+1}(0) = 0 \tag{2.49}$$

for the expansion coefficients $a_{\ell,m}(0)$ appearing in equation (2.48). The (unique) solution of equation (2.49), subject to the boundary condition

$$a_{\ell,0}(0) = \frac{(\ell-1)!}{2^{2\ell+3}}[1 + (-1)^\ell] \tag{2.50}$$

derived in section 2.5.1 (see equation (2.45)), reads ($\ell + m > 0$):

$$a_{\ell,m}(0) = \frac{(\ell+m-1)!}{2^{2(\ell+m)+3}} \left[\left(\frac{f_\beta}{2}+1\right)^m + (-1)^\ell \left(\frac{f_\beta}{2}-1\right)^m \right]. \tag{2.51}$$

Combined with equation (2.48), it yields the leading term in the $1/n$ expansion for joint cumulants of Landauer conductance and the noise power:

$$\delta\kappa_{\ell,m} \simeq \frac{1}{8} \frac{(\ell+m-1)!}{(4n)^{\ell+m}} \left[\left(\frac{f_\beta}{2}+1\right)^m + (-1)^\ell \left(\frac{f_\beta}{2}-1\right)^m \right]. \tag{2.52}$$

Equations (2.47) and (2.52) are the central result of this section.

In particular, it brings the following large-n expression for the cumulants of noise power in the case of symmetric leads:

$$
\langle\langle \mathcal{P}^\ell \rangle\rangle \simeq (G_0\theta)^\ell \Bigg[2n\left(1 + \frac{f_\beta}{4}\right)\delta_{\ell,1} + \left(1 + \frac{f_\beta^2}{8}\right)\delta_{\ell,2}
$$
$$
+ \frac{(\ell-1)!}{8n^\ell}\left[\left(\frac{f_\beta}{2} - 1\right)^\ell + \left(\frac{f_\beta}{2} + 1\right)^\ell\right]\Bigg]. \tag{2.53}
$$

Explicit dependence of the noise power cumulants on both the temperature θ and the bias voltage υ enters through a single function f_β (see equation (1.7)) that depends on the ratio $\beta = \upsilon/2\theta$. Based on equation (2.53), it can further be shown that small but nonvanishing cumulants of the third and higher order are responsible for long exponential tails in the otherwise Gaussian distribution of the noise power (compare with Vivo *et al* 2008).

3. Conclusions

In summary, we have presented an advanced formulation of the recently proposed integrable theory of quantum transport (Osipov and Kanzieper 2008) to study statistics of noise power fluctuations in a chaotic cavity with broken time-reversal symmetry in the crossover regime $(\theta, 0) \rightarrow (0, \upsilon)$ between thermal and shot noise. By *relating the cumulants of noise power to those of the Landauer conductance*, we determined dependence of the noise power cumulants (as well as of joint cumulants of Landauer conductance and the noise power) on the bias voltage υ, temperature θ and the number of channels $N_{L,R}$ in the leads attached to a cavity through ballistic point contacts.

We are confident that ideas of integrability combined with the scattering matrix approach are able to provide a nonperturbative description of many more transport phenomena in chaotic cavities. Quantum transport in cavities with losses (Doron *et al* 1991, Beenakker and Brouwer 2001, Simon and Moustakas 2006) and non-ideal leads (Brouwer 1995) are just two examples of chaotic scattering systems whose detailed study is much called for.

Note added. Recently, we learnt about the paper by Khoruzhenko *et al* (2009) where an alternative approach was developed to describe statistics of conductance and shot-noise power in chaotic cavities with and without time-reversal symmetry. Possibly triggered by the earlier paper by Novaes (2008), these authors combine a theory of the Selberg integral with the theory of symmetric functions to evaluate the (joint) moments of Landauer conductance and the shot-noise power in terms of series over all partitions of the moment's order. In particular, Khoruzhenko *et al* (2009) confirm our large-n formulae equations (2.46) and (2.53) at zero temperature.

Acknowledgments

Appendix A. Cumulant generating function for the Landauer conductance and the fifth Painlevé transcendent

Relation to a gap formation probability. The 'simplest' (albeit not operative) way to observe that the conductance cumulant generating function $\mathcal{F}_n(z, 0)$ (equation (2.10)) can be expressed in terms of Painlevé V is to spot that $\mathcal{F}_n(z, 0)$ is essentially the gap formation probability within the interval $(z, +\infty)$ in the spectrum of an $n \times n$ Laguerre unitary ensemble. Indeed, the transformation of integration variables $\lambda_j = zT_j$ in equation (2.10) yields

$$\mathcal{F}_n(z, 0) = c_n^{-1} z^{-n(n+\nu)} \int_{(0,z)^n} \prod_{j=1}^{n} \mathrm{d}\lambda_j \lambda_j^{\nu} \, \mathrm{e}^{-\lambda_j} \Delta_n^2(\boldsymbol{\lambda}). \tag{A.1}$$

A nonperturbative evaluation of the above n-fold integral is readily available (Tracy and Widom 1994, Forrester and Witte 2002) eventually resulting in the following Painlevé V representation (Osipov and Kanzieper 2008):

$$\mathcal{F}_n(z, 0) = \exp\left(\int_0^z \mathrm{d}t \frac{\sigma_n(t) - n(n + \nu)}{t} \right). \tag{A.2}$$

Here, $\sigma_n(t)$ satisfies the Jimbo–Miwa–Okamoto form of the Painlevé V equation (Jimbo *et al* 1980, Okamoto 1987):

$$(t\sigma_n'')^2 + [\sigma_n - t\sigma_n' + 2(\sigma_n')^2 + (2n + \nu)\sigma_n']^2 + 4(\sigma_n')^2(\sigma_n' + n)(\sigma_n' + n + \nu) = 0 \tag{A.3}$$

subject to the boundary condition $\sigma_{\mathrm{V}}(t \to 0) \simeq n(n+\nu)$. Keeping in mind the parameterization $n = \min(N_{\mathrm{L}}, N_{\mathrm{L}})$ and $\nu = |N_{\mathrm{L}} - N_{\mathrm{R}}|$, one concludes that equation (A.3) is equivalent to equation (2.27) announced in section 2.2.3.

Direct evaluation of $\mathcal{F}_n(z, 0)$. To directly evaluate $\mathcal{F}_n(z, 0)$ defined by equation (2.10), we introduce the associated τ function

$$\tau_n(\boldsymbol{t}; z) = \frac{1}{n!} \int_{(0,1)^n} \prod_{j=1}^{n} \mathrm{d}T_j T_j^{\nu} \mathrm{e}^{-zT_j + V(t; T_j)} \Delta_n^2(\boldsymbol{T}) \tag{A.4}$$

such that

$$\mathcal{F}_n(z, 0) = \frac{n!}{c_n} \tau_n(\boldsymbol{t}; z)\Big|_{t=\boldsymbol{0}}, \tag{A.5}$$

and make use of the KP equation (2.15),

$$\left(\frac{\partial^4}{\partial t_1^4} + 3\frac{\partial^2}{\partial t_2^2} - 4\frac{\partial^2}{\partial t_1 \partial t_3} \right) \log \tau_n(\boldsymbol{t}; z) + 6\left(\frac{\partial^2}{\partial t_1^2} \log \tau_n(\boldsymbol{t}; z) \right)^2 = 0, \tag{A.6}$$

supplemented by the Virasoro constraints[8]

$$[\hat{L}_{q+1}(\boldsymbol{t}) - \hat{L}_q(\boldsymbol{t})]\tau_n(\boldsymbol{t}; z) = 0, \qquad q \geqslant 0, \tag{A.7}$$

where a set of differential operators

$$\hat{L}_q(\boldsymbol{t}) = \hat{\mathcal{L}}_q(\boldsymbol{t}) - z\frac{\partial}{\partial t_{q+1}} + \nu\frac{\partial}{\partial t_q} \tag{A.8}$$

involves the Virasoro operators (equation (2.19)) (the convention $\partial/\partial t_0 \equiv n$ is assumed).

In order to project the KP equation (A.6) onto $\boldsymbol{t} = \boldsymbol{0}$, we need only two Virasoro constraints labelled by $q = 0$,

$$\left[\sum_{j=1}^{\infty} jt_j \left(\frac{\partial}{\partial t_{j+1}} - \frac{\partial}{\partial t_j} \right) - z\frac{\partial}{\partial t_2} + (2n + \nu + z)\frac{\partial}{\partial t_1} \right] \log \tau_n(\boldsymbol{t}; z) = n(n + \nu), \tag{A.9}$$

---

[8] Equations (A.7) and (A.8) readily follow from equations (2.17) and (2.18) upon setting $f_\beta = 0$ and $w = 0$.

and $q = 1$

$$\left[\sum_{j=1}^{\infty} j t_j \left(\frac{\partial}{\partial t_{j+2}} - \frac{\partial}{\partial t_{j+1}} \right) - z \frac{\partial}{\partial t_3} + (2n + \nu + z) \frac{\partial}{\partial t_2} \right.$$
$$\left. - (2n + \nu) \frac{\partial}{\partial t_1} + \frac{\partial^2}{\partial t_1^2} \right] \log \tau_n(\boldsymbol{t}; z) + \left(\frac{\partial}{\partial t_1} \log \tau_n(\boldsymbol{t}, z) \right)^2 = 0. \quad \text{(A.10)}$$

Here

$$\frac{\partial}{\partial t_1} \tau_n(\boldsymbol{t}; z) = -\frac{\partial}{\partial z} \tau_n(\boldsymbol{t}; z). \quad \text{(A.11)}$$

Lengthy but straightforward manipulations with equations (A.9) and (A.10) as well as with their derivatives over t_1 and t_2 projected onto $\boldsymbol{t} = \boldsymbol{0}$ result in equations (2.25) and (2.26).

References

Adler M, Shiota T and van Moerbeke P 1995 Random matrices, vertex operators and the Virasoro algebra *Phys. Lett.* A **208** 67

Adler M and van Moerbeke P 1995 Matrix integrals, Toda symmetries, Virasoro constraints, and orthogonal polynomials *Duke Math. J.* **80** 863

Adler M and van Moerbeke P 2001 Hermitian, symmetric and symplectic random ensembles: PDE's for the distribution of the spectrum *Ann. Math.* **153** 149

Agam O, Aleiner I and Larkin A 2000 Shot noise in chaotic systems: 'classical' to quantum crossover *Phys. Rev. Lett.* **85** 3153

Baranger H U and Mello P A 1994 Mesoscopic transport through chaotic cavities: a random S-matrix theory approach *Phys. Rev. Lett.* **73** 142

Beenakker C W J 1997 Random matrix theory of quantum transport *Rev. Mod. Phys.* **69** 731

Beenakker C W J and Brouwer P W 2001 Distribution of the reflection eigenvalues of a weakly absorbing chaotic cavity *Physica* E **9** 463

Blanter Ya M and Büttiker M 2000 Shot noise in mesoscopic conductors *Phys. Rep.* **336** 1

Blanter Ya M and Sukhorukov E V 2000 Semiclassical theory of conductance and noise in open chaotic cavities *Phys. Rev. Lett.* **84** 1280

Blümel R and Smilansky U 1990 Random-matrix description of chaotic scattering: semiclassical approach *Phys. Rev. Lett.* **64** 241

Brouwer P W 1995 Generalized circular ensemble of scattering matrices for a chaotic cavity with non-ideal leads *Phys. Rev.* B **51** 16878

Büttiker M 1990 Scattering theory of thermal and excess noise in open conductors *Phys. Rev. Lett.* **65** 2901

Büttiker M 1992 Scattering theory of current and intensity noise correlations in conductors and wave guides *Phys. Rev.* B **46** 12485

Chazy J 1911 Sur les équations différentielles du troisième ordre et d'ordre supérieur dont l'intégrale générale a ses points critiques fixes *Acta Math.* **34** 317

Cosgrove C M and Scoufis G 1993 Painlevé classification of a class of differential equations of the second order and second degree *Stud. Appl. Math.* **88** 25

Cron R, Goffman M F, Esteve D and Urbina C 2001 Multiple-charge-quanta shot noise in superconducting atomic contacts *Phys. Rev. Lett.* **86** 4104

Date E, Kashiwara M, Jimbo M and Miwa T 1983 Transformation groups for soliton equations in *Nonlinear Integrable Systems—Classical Theory and Quantum Theory* ed M Jimbo and T Miwa (Singapore: World Scientific)

Doron E, Smilansky U and Frenkel A 1991 Chaotic scattering and transmission fluctuations *Physica* D **50** 367

Flindt C, Fricke C, Hohls F, Novotný T, Netočný K, Brandes T and Haug R J 2009 Universal oscillations in counting statistics *Proc. Natl Acad. Sci. USA* **106** 10116

Forrester P J 2006 Quantum conductance problems and the Jacobi ensemble *J. Phys. A: Math. Gen.* **39** 6861

Forrester P J and Witte N S 2002 Application of the τ function theory of Painlevé equations to random matrices: PV, PIII, the LUE, JUE and CUE *Commun. Pure Appl. Math.* **55** 679

Hua L K 1963 *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains* (Providence, RI: American Mathematical Society)

Imry Y 2002 *Introduction to Mesoscopic Physics* (New York: Oxford University Press)

Jimbo M, Miwa T, Mori Y and Sato M 1980 Density matrix of an impenetrable Bose gas and the fifth Painlevé transcendent *Physica* D **1** 80

Kanzieper E 2002 Replica field theories, Painlevé transcendents, and exact correlation functions *Phys. Rev. Lett.* **89** 250201

Kanzieper E 2005 Exact replica treatment of non-Hermitean complex random matrices in: *Frontiers in Field Theory* ed O Kovras (New York: Nova Science)

Kanzieper E 2009 Replica approach in random matrix theory arXiv:0909.3198 to appear in *The Oxford Handbook of Random Matrix Theory,* ed G Akemann, J Baik and P Di Francesco (Oxford: Oxford University Press)

Khlus V A 1987 Current and voltage fluctuations in micro-junctions of normal and superconducting metals *Sov. Phys.—JETP* **66** 1243

Khoruzhenko B A, Savin D V and Sommers H-J 2009 Systematic approach to statistics of conductance and shot-noise in chaotic cavities *Phys. Rev.* B **80** 125301

Lesovik G B 1989 Excess quantum noise in 2D ballistic point contacts *JETP Lett.* **49** 592

Lewenkopf C H and Weidenmüller H A 1991 Stochastic versus semiclassical approach to quantum chaotic scattering *Ann. Phys. (NY)* **212** 53

Martin T 2005 Noise in mesoscopic physics *Nanophysics: Coherence and Transport* ed H Bouchiat, Y Gefen, S Guéron, G Montambaux and J Dalibard (Amsterdam: Elsevier)

Martin T and Landauer R 1992 Wave-packet approach to noise in multichannel mesoscopic systems *Phys. Rev.* B **45** 1742

Mehta M L 2004 *Random Matrices* (Amsterdam: Elsevier)

Mello P A and Baranger H U 1999 Interference phenomena in electronic transport through chaotic cavities: an information-theoretic approach *Waves Random Media* **9** 105

Mironov A and Morozov A 1990 On the origin of Virasoro constraints in matrix models: Lagrangian approach *Phys. Lett.* B **252** 47

Morozov A 1994 Integrability and matrix models *Physics-Uspekhi* **37** 1

Müller S, Heusler S, Braun P and Haake F 2007 Semiclassical approach to chaotic quantum transport *New J. Phys.* **9** 12

Novaes M 2008 Statistics of quantum transport in chaotic cavities with broken time-reversal symmetry *Phys. Rev.* B **78** 035337

Oberholzer S, Sukhorukov E V and Schönenberger C 2002 Crossover between classical and quantum shot noise in chaotic cavities *Nature* **415** 765

Oberholzer S, Sukhorukov E V, Strunk C, Schönenberger C, Heinzel T and Holland M 2001 Shot noise by quantum scattering in chaotic cavities *Phys. Rev. Lett.* **86** 2114

Okamoto K 1987 Studies on the Painlevé equations: II. Fifth Painlevé equation PV *Jpn. J. Math.* **13** 47

Osipov V Al and Kanzieper E 2007 Are bosonic replicas faulty? *Phys. Rev. Lett.* **99** 050602

Osipov V Al and Kanzieper E 2008 Integrable theory of quantum transport in chaotic cavities *Phys. Rev. Lett.* **101** 176804

Osipov V Al and Kanzieper E 2009 Correlations of RMT characteristic polynomials and integrability: I. Hermitian matrices (in preparation)

Politzer H D 1989 Random-matrix description of the distribution of mesoscopic conductance *Phys. Rev.* B **40** 11917

Richter K and Sieber M 2002 Semiclassical theory of chaotic quantum transport *Phys. Rev. Lett.* **89** 206801

Savin D V and Sommers H J 2006 Shot noise in chaotic cavities with an arbitrary number of open channels *Phys. Rev.* B **73** R081307

Savin D V, Sommers H-J and Wieczorek W 2008 Nonlinear statistics of quantum transport in chaotic cavities *Phys. Rev.* B **77** 125332

Simon S H and Moustakas A L 2006 Crossover from conserving to lossy transport in circular random matrix ensembles *Phys. Rev. Lett.* **96** 136805

Tracy C A and Widom H 1994 Fredholm determinants, differential equations and matrix models *Commun. Math. Phys.* **163** 33

Tu M H, Shaw J C and Yen H C 1996 A note on integrability in matrix models *Chin. J. Phys.* **34** 1211

Vivo P, Majumdar S N and Bohigas O 2008 Distributions of conductance and shot noise and associated phase transitions *Phys. Rev. Lett.* **101** 216809